

# Modeling the forage harvester logistics process for agricultural resource planning

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Agricultural contractors often face the complex planning problem of having to optimize the utilization of their heterogeneous vehicle fleet. In this paper, we present a mathematical model that optimizes of the logistics processes of silo corn harvest and slurry application. The model simulates the use of primary vehicles (forage harvesters or slurry spreaders) in the field, whereby their utilization rate depends on the support vehicles assigned to them (crop or slurry transport vehicles) and the distance between the field and the silo. Using real data of the corn harvest of an agricultural cooperative in Brandenburg, we show that practical problems can be solved with a software-based approach using mixed-integer programming. We investigate different planning scenarios in order to calculate, for example, the time savings with a greater vehicle fleet or with a more powerful forage harvester.

## Keywords

Logistics, disposition, corn harvest, discrete optimisation

Weather conditions, hardly predictable events, and the legal situation complicate the scheduling of their agricultural machinery for farmers and contractors, especially when processes are time-critical, such as harvesting or slurry application. Workloads are subject to extreme variability and peak times require a maximum utilization of the agricultural machinery. To be able to guarantee a high utilization rate during peak demand periods such as corn harvesting, sufficient crop transport vehicles must be available to transport the biomass to the silo (see forage harvesters and crop transport vehicles in Figure 1). If there is no crop transport vehicle available temporarily to transfer the biomass from the forage harvester, expensive downtimes occur. Manual (non-automated) planning is particularly time-consuming when several forage harvesters are in operation and the required number of crop transport vehicles per field and harvester varies due to different field-silo distances. The problem can also be adapted to other processes, such as slurry application with slurry spreaders and slurry transport vehicles. Here, the distance between silo and field and the available slurry transport vehicles determine the utilization of the spreader, analogously to the utilization of the forage harvester. In contrast to the studies of MEDERLE and BERNHARDT (2017) and JENSEN et al. (2012), in which the route of the vehicles in the field is the focus of the study, we take the route of vehicles within fields as given and the distance between forage harvester and silo is assumed to be constant.



Figure 1: Forage harvesters and crop transport vehicles (© Planitz)

In this paper, we investigate the harvest process of one or more forage harvesters supported by several crop transport vehicles. Transport vehicles receive the biomass directly from the forage harvester and transport it to the silo. The goal of this work is to formulate the constraint programming (CP) model presented by BENDER et al. (2021) for dispatching different types of vehicles as a mathematical optimization model. Furthermore, the corn harvest of an agricultural cooperative in Brandenburg is investigated as a case study, which requires an extension of the model and serves for validation.

The focus is on the interaction of forage harvesters and crop transport vehicles. To simplify the planning problem, we do not model the behavior and number of compaction vehicles in detail. However, they are considered implicitly by the parameter unloading time of the transport vehicles at the silo. We solve the modeled optimization problem with a commercial mixed-integer programming solver to generate suitable schedules and to investigate process alternatives with respect to the number and characteristics of the vehicles deployed. For this purpose, an objective function with constraints represents the logistic processes.

AMIAMA et al. (2015a) addressed a similar problem: They developed a simulation model to schedule harvesting processes. However, simulation does usually not produce a mathematically optimal solution. AMIAMA et al. (2015b) evaluated the cost-dependent marginal utility of deployed transport vehicles for individual fields. Although routing between fields for forage harvesters is performed in this paper, no transport vehicle scheduling performed. CEDEIRA-PENA et al. (2017) minimized the routing of forage harvesters over multiple periods. However, variability in processing time as a function of transport vehicle number was not considered. There exist several studies regarding forage harvester route planning, however, integrated analytical scheduling of forage harvesters and transport vehicles, where the number of transport vehicles determines forage harvester utilization, has not yet been applied to a practical problem.

## Method

Mixed-integer programming can solve a wide variety of practical problems, such as the creation of train schedules, production planning, or routes for shipping companies or parcel services. In contrast to linear programming, all or some variables can only take integer values. A typical solution method

to find the optimum of these problems are branch & bound based algorithms. A software based on these algorithms is called solver, such as Gurobi, CPLEX or GLPK. For this purpose, an objective function with constraints is defined in order to span and limit a solution space. In the solution process, a usually non-integer initial solution (relaxation) is determined for each iteration by the simplex algorithm. This is followed by branching into the neighboring integer values in order to calculate the (again, usually non-integer) values of further integer variables of the problem. Complex problems require numerous iterations of the algorithm, whereby the solver regularly compares the best solution found with the solutions of the relaxations in order to estimate how far it is from the theoretically possible optimum. These solutions serve as bounds (upper and lower) and approach each other in the course of the iterations until the upper and lower bounds have the same value. Then, optimality is proven and the algorithm terminates. If the algorithm terminates prematurely because it exceeds the maximum computing time, the distance between the bounds, known as the gap, is output as a percentage. This indicates the maximum distance between the best solution found and the theoretical non-integer optimum. The number of iterations depends largely on the type of modeling. Modern solvers also use additional heuristics to find solutions more quickly.

For more details on the procedure, we refer to WOLSEY (1998). In the field of agriculture, mixed-integer programming has been used in determining the optimal mix of crop (FILIPPI et al. 2017), the optimal distribution of agricultural machinery to different farms (CAMARENA et al. 2002) and the optimal transport of fruit to the processing plant. (LAMSAL et al. 2016).

### Example

The CP Model presented by BENDER et al. (2021) is modeled as a fully-directed graph and represents the movements of two different types of vehicles (primary and support). Primary vehicles require one or more support vehicles to fulfill a task. The number of support vehicles and node-specific properties determine the processing time at the nodes. The number of support vehicles required per primary vehicle varies from node to node. This abstract model can be transferred to the harvesting process with forage harvesters and transport vehicles: While forage harvesters (primary vehicles) process different fields (nodes), they are supported by transport vehicles (support vehicles). The number of transport vehicles required to fully utilize the forage harvesters depends on the distance between the field and the silo. A longer distance means longer travel times and, as a result, requires more transport vehicles to achieve a high capacity utilization of the forage harvester. If fewer transport vehicles are available, the utilization of the forage harvester decreases and the processing time of a field increases accordingly. The model, therefore, does not represent the individual journeys of the transport vehicles between the silo and the field itself, but their effect on the utilization of the forage harvester in the field.

Figure 2 provides an example of an optimized route for five fields, two forage harvesters, and six transport vehicles for randomly generated field locations. Sizes and number of transport vehicles for full utilization of forage harvesters per field is also shown. The tables at each node show the processing times (in hours) for different numbers of transport vehicles with a single forage harvester. The actual processing time is printed in bold.

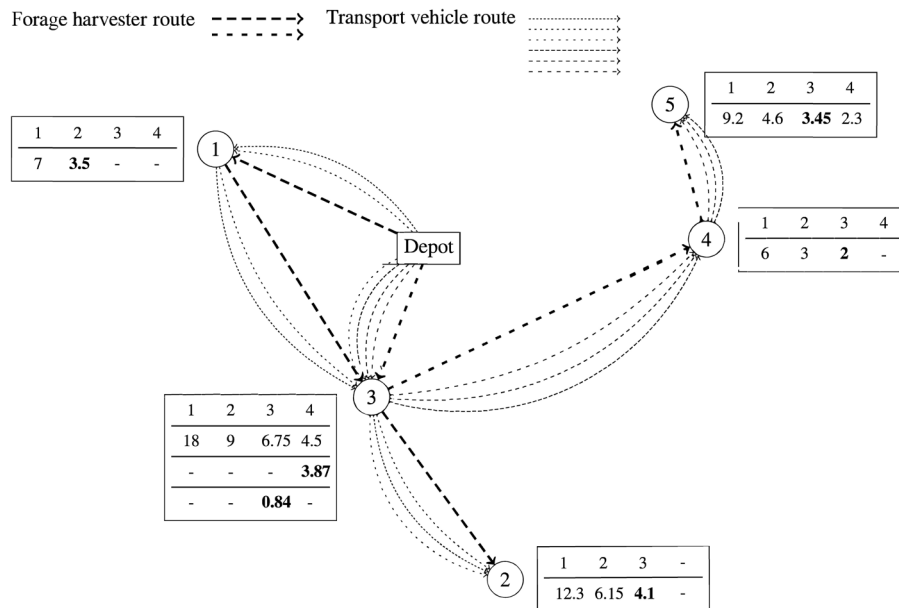


Figure 2: Example of an optimal schedule with randomly generated data for five fields, two forage harvesters and six transport vehicles. The tables at each node show the processing times (in hours) for different numbers of transport vehicles with a single forage harvester. The actual processing time is printed in bold

Two forage harvesters start here from the depot, with forage harvester A traveling to field 3 together with four transport vehicles while forage harvester B travels to field 1 with two transport vehicles. However, field 3 is not completely processed by forage harvester A in the optimum solution. Instead, it moves on to fields 4 and 5 with only three transport vehicles. After forage harvester B completely processed field 1, forage harvester B travels from field 1 to field 3 in order to finish the processing there together with the transport vehicle left behind by forage harvester A. Finally, forage harvester B and the three transport vehicles drive to field 2 to finish the harvest.

For the practical application to the case study discussed in this paper, the following extensions are necessary:

- Scheduling is done over several periods (days), with vehicles returning to the depot at the end of each period and starting from there in the next period.
- Forage harvesters are not identical to each other, which means that forage harvesters may differ in terms of working speed (depending on working width and power).
- The relationship between the number of transport vehicles and the utilization of the forage harvester is not continuously linear (i.e. the relationship between the number of transport vehicles and the processing time is not continuously inversely proportional).

## Notations

In this work, we transform the CP model presented by BENDER et al. (2021) and extend it to meet the practical planning problem requirements. Firstly, we divide the planning period into periods. In each period, vehicles leave the depot at most once and return to it, corresponding to one shift or one working day. In the original model, the goal is to minimize the total harvest time (Makespan). Since the working hours per period and the number of periods available are given, we deploy a cost function. The cost function adds up the machine hours of all forage harvesters, consisting of processing times and travel times between fields. An extension by further factors (e.g. machine hour rate, personnel costs) is possible with little effort.

The problem is modeled as a complete direct graph. Fields are represented as nodes, edges correspond to travel times between two nodes. Since a field can be served by multiple primary vehicles, each field is represented by as many overlapping nodes as there are primary vehicles. Each node is thus approached by at most one primary vehicle. Each primary vehicle, therefore, has its own subgraph in which it can move exclusively. Support vehicles can move between primary vehicles, i.e. they are not restricted to the subgraphs. The sets used in the mathematical model are given in Table 1.

Table 1: Sets used in the mathematical formulation

Set	Definition
$P$	Set of all primary vehicle. Example: two forage harvesters $\Rightarrow P = \{1, 2\}$ .
$S$	Set of all support vehicles. Example: four support vehicles $\Rightarrow S = \{1, 2, 3, 4\}$ .
$F$	Set of all fields. Example: five fields $\Rightarrow F = \{1, 2, 3, 4, 5\}$ .
$K$	Set of all field nodes. For each primary vehicles that may visit a field there is a duplicate node. Example: $ K  =  F  \cdot  P  = 5 \cdot 2 = 10 \Rightarrow K = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$K_p$	Subset of field nodes that primary vehicle $p$ may visit. Example: $ K  = 10,  P  = 2 \Rightarrow K_1 = \{1, 2, 3, 4, 5\}, K_2 = \{6, 7, 8, 9, 10\}$ .
$N$	Set of all nodes in the model graph, consisting of field nodes, start depot 0 and end depot $n$ . Start and end depot with same coordinates. $N = 0 \cap K \cap n$ .
$N^+, N^-$	Subset of $N$ . Possible start nodes ( $N^+$ ) and end nodes ( $N^-$ ) for a trip between two nodes. $N^+ = 0 \cap K$ and $N^- = K \cap n$ .
$N_p^+, N_p^-$	Set of start nodes ( $N_p^+$ ) and end nodes ( $N_p^-$ ), which primary vehicle $p$ may visit. $N_p^+ = 0 \cap K_p$ and $N_p^- = K_p \cap n$ .
$\tilde{N}_i$	Set of all field nodes of field $i$ . One node for each primary vehicle, that may visit field $i$ . Example: $ P  = 2,  F  = 5 \Rightarrow \tilde{N}_1 = \{1, 6\}, \tilde{N}_2 = \{2, 7\}$ .
$A_i$	Set of all possible numbers of support vehicles that may visit field node $i$ . Example: A primary vehicles requires for full utilization at field $i$ three support vehicles $\Rightarrow A_i = \{1, 2, 3\}$ .
$D$	Set of periods within the planning horizon. Example: seven workdays $\Rightarrow D = \{1, 2, 3, 4, 5, 6, 7\}$ .

The aim of this work is to create a deterministic model. Therefore, in the mathematical formulation, all key figures of the vehicles, as well as fields ,are assumed to be constant (Table 2).

Table 2: Constants used in the mathematical formulation

Konstante	Definition
$\tau_{ij} \in \mathbb{R}^+$	Travel time of a vehicle from node $i$ to node $j$ .
$f_i \in \mathbb{R}^+$	Area of field $i$ .
$r_i \in \mathbb{R}^+$	Harvest duration of one hectar at primary vehicle specific field node $i$ with full utilization.
$u_i^a \in (0, 1]$	Utilization of a primary vehicle with $a$ support vehicles at field node $i$ .
$T \in \mathbb{R}^+$	Period length.

The utilization of a primary vehicle at field node  $i$  results from the loading times of all support vehicles per cycle in relation to the cycle time of a secondary vehicle (equation 1).

$$u_i^a = \min \left( \frac{a \cdot \tau_L}{\tau_L + \tau_{UL} + 2 \cdot \tau_{is}}, 1 \right) \tag{1}$$

With the number of secondary vehicles  $a$ , the loading time  $\tau_L$ , the unloading time  $\tau_{UL}$  and the travel time  $\tau_{is}$  between field  $i$  and silo. For the forage harvesting process, the times of the phases of the transport cycle and the loading times of the individual transport vehicles at the forage harvester are shown in Figure 3. If the forage harvester is fully utilized, idle times for transport vehicles at the field or forage harvester may occur at the end of its cycle (in the example for  $a = 5$ ). If the forage harvester is not fully utilized, i.e. there are idle times at the forage harvester, the loading process of transport vehicle 1 follows directly after the return to the field or forage harvester without idle times.

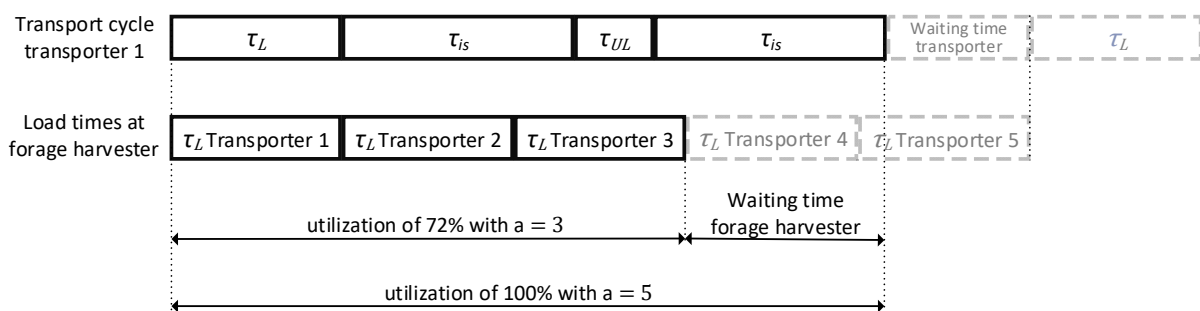


Figure 3: Duration of the phases of the transport cycle of a transport vehicle and loading times of different transport vehicles at the forage harvester

The loading time  $\tau_L$  is derived from the process time  $r$  (in min/ha) of a fully utilized primary vehicle, the loading capacity  $L$  (in  $m^3$ ) of the support vehicles and the volume to be transported  $M$  in  $m^3/ha$  (equation 2).

$$\tau_L = \frac{r \cdot L}{M} \tag{2}$$

In this model, we assume that the biomass volume per hectare  $M$  and the load capacity  $L$  are constant. From a yield per hectare of  $50 \text{ t/ha}$  and a relative crop mass of  $0,3 \text{ t/m}^3$  results  $M = \frac{50 \text{ t/ha}}{0,3 \text{ t/m}^3} = 166,7 \text{ m}^3/\text{ha}$ . If several primary vehicles are in use,  $r$  can vary between them.

Figure 4 illustrates the harvester utilization for different travel times between silo and field and different numbers of transport vehicles. In the example, depending on the travel time between the field and silo, two to four transport vehicles are required for full utilization of a forage harvester. The relationship between the number of transport vehicles and the utilization of the forage harvester is initially linear. However, the last transport vehicle can often only increase the utilization of the forage harvester by a smaller value than the previous transport vehicles, i.e. the marginal utility of the last transport vehicle is lower. For example, while at a silo-field travel time of 9.2 minutes the fourth transport vehicle raises the utilization from about 78 to 100%, at a silo-field travel time of 7.1 minutes it raises the utilization only from about 92 to 100%. Thus, in the latter case, the marginal utility of the fourth transport vehicle is lower. If several forage harvesters are in operation and the number of transport vehicles is limited, forage harvesters thus compete for available transport vehicles. The transport vehicle schedule can thus have a major influence on the processing time of a field.

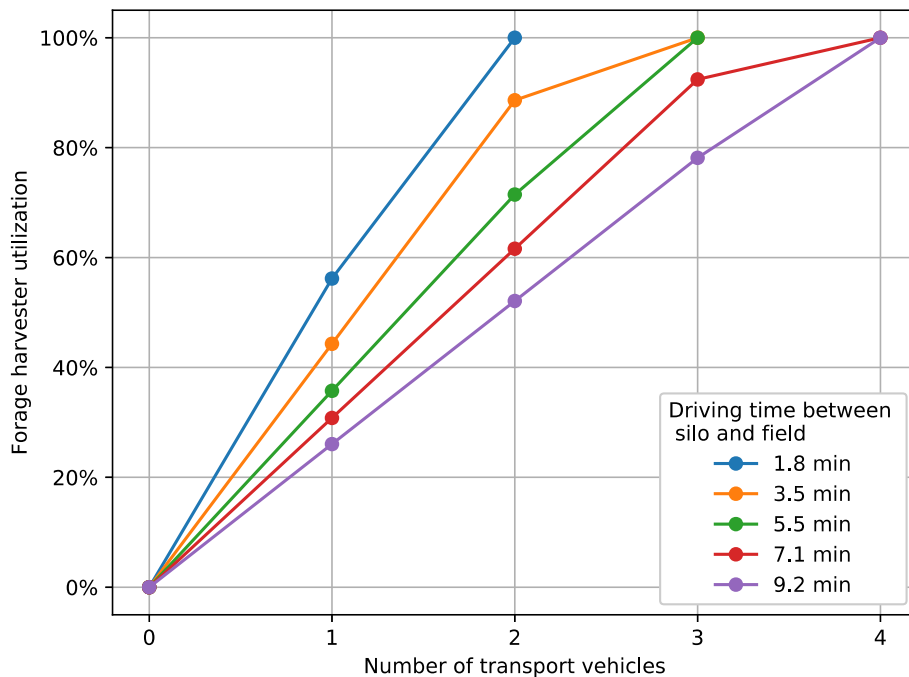


Figure 4: Relation between forage harvester utilization ( $p$ ) and number of transport vehicles ( $m$ ) on a field for various driving times between field and silo (with  $M = 166,7 \text{ m}^3/\text{ha}$ ,  $L = 40 \text{ m}^3$ ,  $r = 30 \text{ min/ha}$ ,  $t_{UL} = 2 \text{ min}$ ,  $\tau_{IS}$ : see legend)

Table 3 defines the binary, integer and continuous variables in the mathematical model.

Table 3: Variables used in the mathematical formulation

Variable	Definition
$x_{ij}^d \in \{0,1\}$	1, if a primary vehicle travels from node $i$ to node $j$ in period $d$ , else 0.
$q_i^d \in \{0,1\}$	1, if a primary vehicle visits node $i$ in period $d$ , else 0.
$y_i^{ad} \in \{0,1\}$	1, if a primary vehicles services node $i$ with a number of $a$ support vehicles in period $d$ , else 0.
$v_{ij}^d \in \{0,1\}$	1, if one or more support vehicles travel from node $i$ to node $j$ in period $d$ , else 0.
$w_{ij}^d \in \mathbb{N}$	Number of support vehicles that travel from node $i$ to node $j$ in period $d$ .
$t_i^d \in \mathbb{R}^+$	Service start time at node $i$ in period $d$ . 0, if no vehicle visits node $i$ in period $d$ .
$s_i^{ad} \in \mathbb{R}^+$	Service duration at node $i$ in period $d$ with a number of $a$ support vehicles.

### Objective function and constraints

We split the constraints of the mathematical into three categories: the constraints are either associated with primary vehicles movement, support vehicle movement or the primary vehicle utilization. The objective function (3) minimizes the sum over the service times  $s_j^{ad}$  and travel times  $\tau_{ij}$  between all field nodes  $F$  for all periods  $D$ .

$$\min \sum_{j \in F} \sum_{a \in A_j} \sum_{d \in D} s_j^{ad} + \sum_{i \in N} \sum_{j \in N, j \neq i} \sum_{d \in D} \tau_{ij} \cdot x_{ij}^d \tag{3}$$

Constraints (4) to (9) describe the movement of primary vehicles. Constraints (4) ensure that all primary vehicles of set  $P$  leave the depot 0 in each period not more than once to enter their designated subset of nodes  $K_p$ . Constraints (5) and (6) are the flow conservation constraints and ensure that primary vehicles leave a node that they visit in the same period. Constraints (7) ensure that the primary vehicles visit one or more nodes that represent a single field (i.e. every field is visited at least once). Constraints (8) calculate the service starting time  $t_j^d$  at node  $j$  summing up the start time of the previous node, its processing time and the travel time between nodes. If node  $i$  is not the predecessor of node  $j$ ,  $x$  is equals 0 and the negative term ensures that the inequality is still valid. Constraints (9) enforce that the service start time is 0 if no vehicles visit node  $i$ .

$$\sum_{j \in K_p} x_{0j}^d \leq 1 \quad \forall p \in P, \forall d \in D \tag{4}$$

$$\sum_{j \in N_p^+} x_{ij}^d \leq q_j^d \quad \forall p \in P, \forall j \in K_p, \forall d \in D \tag{5}$$



$$\sum_{j \in N_p^-} x_{ji}^d \leq q_j^p \quad \forall p \in P, \forall j \in K_p, \forall d \in D \quad (6)$$

$$\sum_{i \in \bar{N}_j} \sum_{d \in D} q_i^d \geq 1 \quad \forall j \in F \quad (7)$$

$$t_j^d \geq t_i^d + \sum_{a \in A_i} s_i^{ad} + \tau_{ij} \cdot x_{ij}^d - T \cdot (1 - x_{ij}^d) \quad (8)$$

$$\forall p \in P, \forall i \in N_p^+, \forall j \in N_p^-, \forall d \in D$$

$$t_j^d \leq T \cdot \sum_{i \in N_p} x_{ij}^d \quad \forall p \in P, \forall j \in N_p^-, \forall d \in D \quad (9)$$

Constraints (10) to (14) define the movement of support vehicles. Constraints (10) limit the number of support vehicles leaving the depot in period  $d$  to the number of support vehicles available  $S$ . Constraints (11) are the flow conservation constraints and ensure that the number of support vehicles visiting a node also leaves it. Constraints (12) and (13) connect the binary decision variables  $v$  with the integer decision variables  $w$  that the following applies:  $w = 0 \Rightarrow v = 0$  und  $w \geq 1 \Rightarrow v = 1$ . Constraints (14) calculate the service starting time analogously to (8).

$$\sum_{j \in N^-} w_{0j}^d \leq |S| \quad \forall d \in D \quad (10)$$

$$\sum_{h \in N^+} w_{hi}^d = \sum_{j \in N^-} w_{ij}^d \quad \forall i \in K, \forall d \in D \quad (11)$$

$$w_{ij}^d \leq v_{ij}^d \cdot |S| \quad \forall i \in N^+, \forall j \in N^-, \forall d \in D \quad (12)$$

$$w_{ij}^d \geq v_{ij}^d \quad \forall i \in N^+, \forall j \in N^-, \forall p \in P \quad (13)$$

$$t_j^d \geq t_i^d + \sum_{a \in A_i} s_i^{ad} + \tau_{ij} \cdot v_{ij}^d - T \cdot (1 - v_{ij}^d) \quad (14)$$

$$\forall i \in N^+, \forall j \in N^-, \forall d \in D$$

Constraints (15) to (18) set the support vehicle utilization considering the number of support vehicles associated with the harvester. Constraints (15) define, that a primary vehicle services a field in exactly one utilization (mode). Constraints (16) ensure that the number of support vehicles supporting a primary vehicle does not exceed the number of vehicles visiting the node. Constraints (17) calculate the service time  $s_j^{ad}$  for a single primary vehicle on field  $j$  with the number of support vehicles  $a$ , the primary vehicle utilization  $u_j^a$ , the maximum working speed of a primary vehicle  $r_j$  and the field size  $f_j$ . Constraints (18) assure the processing times of all primary vehicles are sufficient to process the entire field.

$$\sum_{a \in A_j} y_j^{ad} = q_j^d \quad \forall j \in K, \forall d \in D \quad (15)$$

$$\sum_{a \in A_i} y_j^{ad} \cdot a = \sum_{i \in N^+} w_{ij}^d \quad \forall j \in K, \forall d \in D \quad (16)$$

$$s_j^{ad} \leq y_j^{ad} \cdot \frac{r_j \cdot f_j}{u_j^a} \quad \forall j \in K, a \in A_j, \forall d \in D \quad (17)$$

$$\sum_{i \in \bar{N}_j} \sum_{a \in A_i} \sum_{p \in P} s_i^{ad} \cdot \frac{u_i^a}{r_i} = f_j \quad \forall j \in F \quad (18)$$

### Case study

The model is validated using corn silage harvest data from an agricultural cooperative in Brandenburg, Germany. The harvest takes place within seven weeks, whereby the allocation of fields to the respective weeks is determined by the date of sowing. For each individual week, the optimal harvesting plan is to be created with regard to the minimum number of machine hours. The number of fields, field sizes, and distances between fields and their associated silo sometimes vary greatly between fields and weeks. The size of the smallest field is less than 1 ha, while that of the largest field is 110 ha. Table 4 shows the cumulative total size and average travel time between field and silo per week. Due to the lack of information on agricultural roads or their passability and traffic (MICHELS et al. 2018), and due to the neglect of field travel times (especially relevant for larger fields), the actual travel times are larger than assumed and non-deterministic. However, a constant, deterministic data basis is sufficient for the investigations planned within the scope of this work, since the modeling and evaluation of machine characteristics is the focus.

Table 4: Number of fields, cumulated field size and average travel time between silo and field for each harvest week

Week	Number of fields	Field size in ha	Average travel time silo-field in min
1	11	149.9	2.7
2	8	161.5	7.0
3	9	161.8	5.3
4	10	146.7	3.7
5	10	166.4	5.5
6	8	189.6	5.0
7	4	135.7	2.2
Total	60	1111.6	4.6

In the following calculations, the working days per week and the working hours per day are assumed to be constant. There are seven working days with two shifts of seven hours each, resulting in a weekly working time of up to 98 hours. In this case study, we observe the value of the objective function for different test settings. For this purpose, we vary the number of vehicles, some vehicle characteristics, such as the power of the chippers and the capacity and unloading time of the transport vehicles, and finally the available working time.

We use Gurobi 9.1 software with Python 3.6 API to solve the model. All computations were performed on an Intel(R) Xeon(R) CPU E5-2680 v3 running at 2.50 GHz with 8 cores and 16 GB of memory, with a maximum computation time of 60 minutes per parameter setting. Travel times between depot, fields, and silos are determined using the Open Route Service Matrix API, which is based on the route network from Open Street Maps.

### Automated harvest schedule creation

Within the specified maximum computing time, the solver finds a solution for all solvable problems with a maximum deviation of 0.6% (gap) from the (unknown) optimal solution of the model. The algorithm finds good solutions very quickly, but takes a long time to prove optimality. For a practical application, which is naturally subject to deviations, which were not taken into account here, the quality of the solutions is considered to be sufficient.

With all constraints taken into account, the solver finds weekly production schedules with minimized forage harvester machine hours. We define machine hours as the sum of the harvesting times on the fields and the travel times between the fields or the depot. Breaks, such as lunch breaks, are not considered in this schedule. Figure 5 shows a harvest schedule for week 1 based on the optimal solution of the model with a single forage harvester. The edges between the fields shown in the graph do not correspond to the actual travel paths, but represent the sequence of field. The Gantt chart represents time and duration of the field harvest. Some bars are directly bordering each other and illustrate the short travel times between some fields. The influence of travel times is significantly greater when fields are highly scattered and smaller, as it is common in southern German states. Then, the consideration of travel times is even more impactful, especially for contractors.

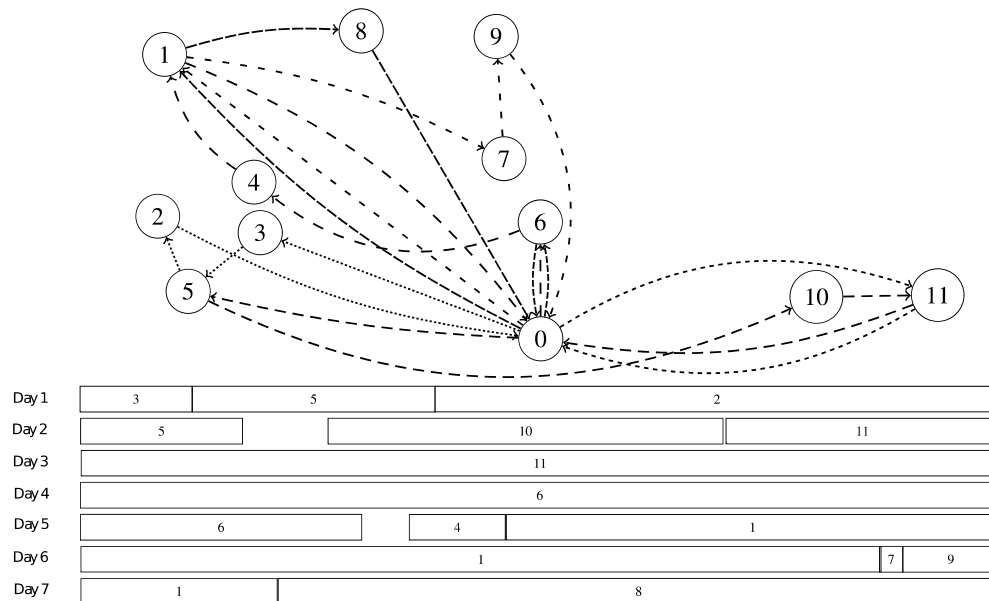


Figure 5: Daily harvest plan for week 1

### Influence of vehicle characteristics on the total working time

The agricultural cooperative under consideration uses a forage harvester which, at full capacity, harvests one hectare of corn in an average of 30 minutes. Tractors with two trailers each are used for transport, which results in a transport capacity of about 40 m<sup>3</sup> per transport vehicle. Unloading both trailers at the silo takes two minutes. In order to investigate the influence of the different parameters, we present further variants below, which differ in parameterizations. The variant with the vehicle types and vehicle characteristics currently used in the agricultural cooperative is called the basic variant for the remainder of this paper, regardless of the number of transport vehicles.

We investigate the following variants within the scope of this work:

- a) Various numbers of transport vehicles
- b) Reduction of the harvest time per hectare of a forage harvester to investigate the influence of a more powerful forage harvester on the total harvest time
- c) Variation of the capacity and the unloading time at the silo of the transport vehicles
- d) Additional forage harvester
- e) Reduction of the maximum processing time available

The effects of a), b) and c) on the number of forage harvester machine hours are shown in Table 5, and the effects of a), d) and e) are shown in Table 6. If a harvesting schedule cannot be established for a certain week, this can be attributed to a too large a number of fields, too great field sizes, or too large field-silo distances and thus the number of transport vehicles needed. For such variants, the number of weekly plans that can be created for the entire seven-week harvest period is noted in the last column, there average harvester machine hours are omitted.

Table 5: Average weekly machine hours of forage harvesters with varying power and varying configurations and numbers of transport vehicles with seven working days of 14 hours maximum working time each

Unload time ( $t_{UL}$ ) in min	Transport capacity (L) in $m^3$	Process time (r) in min/ha	Number of transport vehicles (m)	Average harvester machine hours per week in h	Feasible weekly schedules
2	40	30	2	-	2 von 7
			3	84.3	
			4	80.9	
			5	80.7	
			2	85.3	
	40	24	3	71.3	
			4	65.7	
			5	64.7	
			2	-	1 of 7
			3	-	5 of 7
	30	30	4	82.8	
			5	80.9	
			2	-	2 of 7
			3	85.0	
			4	82.2	
1	30	30	5	80.8	
			2	87.0	
			3	80.8	
			4	80.7	
			5	80.7	

The results of the basic variant illustrate the relevance of the number of transport vehicles employed. With two transport vehicles, harvesting plans could only be generated for two out of seven weeks, since the maximum available weekly working hours were exceeded in the other weeks. Therefore, at least three transport vehicles are required to complete the harvest. A fourth transport vehicle increases forage harvester utilization, reducing average weekly machine hours by 3.4 hours. However, a fifth transport vehicle barely reduces forage harvester machine hours. This decreasing marginal utility is due to a saturation effect at the fields once the maximum number of transport vehicles is reached at some fields.

Reducing the process time for one hectare from 30 to 24 minutes, enables one forage harvester with only two transport vehicles to finish every week during the given daily working hours. The average weekly machine hours when using two transport vehicles are only slightly higher than the machine hours of a forage harvester with a process time of 30 minutes per hectare and three transport vehicles. Thus, a more powerful forage harvester can replace a transport vehicle in this scenario and vice versa. In addition, a higher number of transport vehicles can significantly reduce the average weekly machine hours. Figure 6 depicts the average machine hours per week for two forage harvesters with different process times per hectare. While the minimum number of machine hours is already reached with four transport vehicles for a forage harvester with a process time of 30 minutes per hectare, a fifth transport vehicle can reduce the total machine hours for a process time of 24 minutes per hectare.

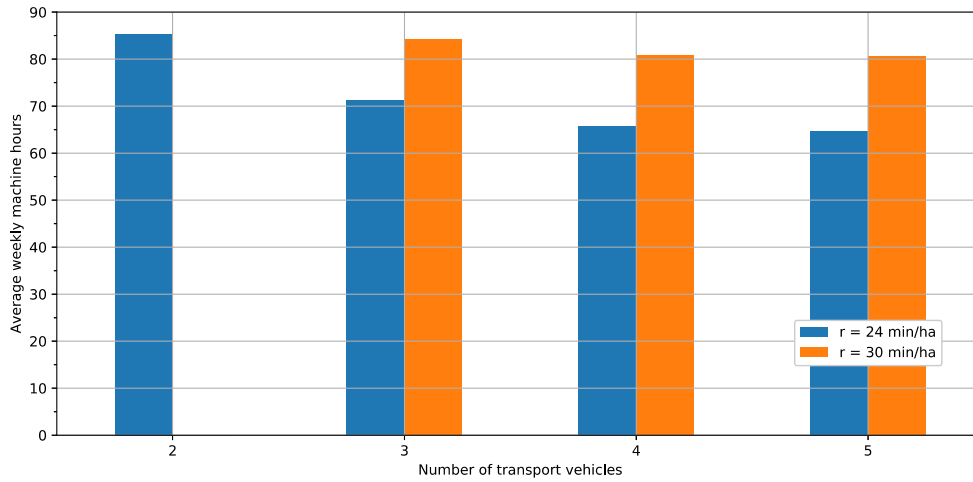


Figure 6: Average weekly working hours depending on the minimum process time (per hectare) of a forage harvester and the number of transport vehicles (if problem is feasible)

The characteristics of employed transport vehicles (capacity and unloading speed) also strongly influence the average weekly machine hours of a forage harvester (Figure 7). Here, we compare different trailer sizes: Two smaller trailers, particularly common in eastern Germany, have a total capacity of 40 m<sup>3</sup>. If a single trailer with a capacity of 30 instead of 40 m<sup>3</sup> is employed and the unloading time remains the same, harvesting schedules cannot be generated for all seven weeks with only three transport vehicles due to the maximum shift times. However, if the unload time is reduced from two to one minute, it is possible to create all seven weekly plans with just three transport vehicles. Transport vehicles with a capacity of 60 m<sup>3</sup> and an unloading time of one minute enable a fleet reduction to only two transport vehicles. With three transport vehicles of this kind, the average machine hours of the forage harvesters are almost at its minimum. Employing five transport vehicles of any type reduces the machine hours to the same value. This is because the minimum machine hours of the forage harvester are already reached. However, this may lead to waiting times for transport vehicles. For further improvement, a more powerful or an additional forage harvester is required.

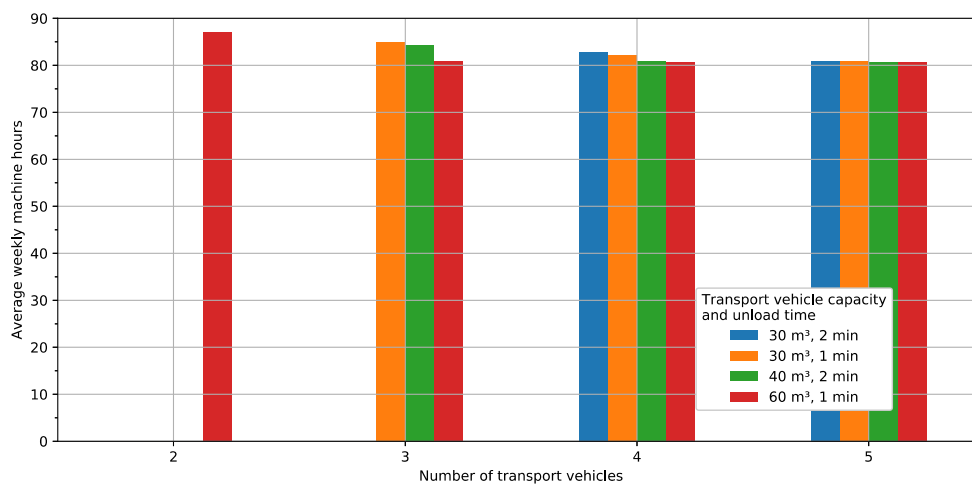


Figure 7: Average weekly machine hours in relation to the transport vehicle capacity, unloading time and number of transport vehicles (if problem is feasible)

Table 6 summarizes operational scenarios with two forage harvesters compared to the variant with only one forage harvester. In addition to the change in the number of forage harvesters and transport vehicles, the available working hours are also reduced. An additional forage harvester cannot reduce the total machine hours of the forage harvesters (the sum of the machine hours of all harvesters is almost the same regardless the forage harvester count). Therefore, if working hours of seven days of 14 hours each are available, it is not reasonable to use another forage harvester due to high investment costs. If, on the other hand, the available working hours are reduced to six days at ten hours each, a second forage harvester is required to completely harvest all fields for each week. It also requires at least six transport vehicles, while the machine hours of a single forage harvester are already at their minimum for with five transport vehicles.

Table 6: Total processing times with varying numbers of forage harvesters and transport vehicles for different maximum working times

Number of forage harvesters	Working days per week	Working hours per day	Number of transport vehicles	Average harvester machine hours per week in h	Feasible weekly schedules
1	7	14	2	-	2 of 7
		14	3	84.3	
		14	4	80.9	
		14	5	80.7	
2	7	14	4	80.9	
		14	5	80.7	
		14	6	80.7	
		14	7	80.7	
2	6	10	4	-	4 of 7
		10	5	-	5 of 7
		10	6	81.3	
		10	7	81.1	

## Conclusions

In this article, we presented a mathematical model for the logistical process of forage harvesting or slurry application by extending an existing model by periods and modifying its objective function. Applying a mixed-integer programming solver, we generated harvest schedules with optimized driving and harvest times of the forage harvesters deployed based on the data of an agricultural cooperative from Brandenburg. In addition, we varied the number and characteristics of the vehicles in order to investigate the influence of different configurations on the processing time. Thus, in addition to the automated generation of time-efficient harvesting schedules, the presented model is suitable for evaluating the deployed machinery for specific problems.

Various extensions for the model presented can be implemented easily, such as weather conditions or other problem-specific influences, which may require that some fields should be processed within certain time windows. An implementation of time windows can even have a possible influence on the computation time (smaller solution space). For problem instances with very small fields or low biomass, a more detailed modeling approach could be valuable, that considers individual journeys of transport vehicles to the silo and back again. This however comes at the expense of computing time. For larger problems - for example, scheduling the entire harvest period of seven weeks instead of

weekly planning - very high computing times are required. For this scenario, we suggest the implementation of a problem-specific heuristics approach.

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